Tutorial #4

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

Problems (Please note these are from Holt's Linear Algebra Text)

2.3 - #27 Calculation Determine if one of the given vectors is in the span of the other vectors.

$$u = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}, \quad v = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}, \quad w = \begin{pmatrix} -5 \\ 7 \\ -7 \end{pmatrix}$$

Solution If suffices to check if the vectors are linearly independent, so we row reduce...

$$\begin{pmatrix} 4 & 3 & -5 \\ -1 & 5 & 7 \\ 3 & -2 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & -5 & -7 \\ 4 & 3 & -5 \\ 3 & -2 & -7 \end{pmatrix} \sim \begin{pmatrix} 1 & -5 & -7 \\ 0 & 23 & 23 \\ 0 & 13 & 14 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

This implies the vectors are linearity independent, so none of the vectors are in the span of the others. □

2.3 #57 Proof Prove that if $\{u_1, u_2, u_3\}$ is a linearly independent set of vectors, then so is $\{u_1 + u_2, u_1 + u_3, u_2 + u_3\}$.

Solution Since $\{u_1, u_2, u_3\}$ are linearly independent, we may assume

$$au_1 + bu_2 + cu_3 = 0 \iff a, b, c = 0$$

Now if we take an arbitrary linear combination of $\{u_1 + u_2, u_1 + u_3, u_2 + u_3\}$ and set it to zero, we see

$$x_1(u_1 + u_2) + x_2(u_1 + u_3) + x_3(u_2 + u_3) = (x_1 + x_2)u_1 + (x_1 + x_3)u_2 + (x_2 + x_3)u_3 = 0$$

Thus, by our assumption we see

$$\begin{cases} x_1 + x_2 = 0 \\ x_1 + x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$$

It's easy to check that the only solution to the above linear system is $x_1, x_2, x_3 = 0$, thus

$$x_1(u_1 + u_2) + x_2(u_1 + u_3) + x_3(u_2 + u_3) = 0 \iff x_1, x_2, x_3 = 0$$

which means $\{u_1 + u_2, u_1 + u_3, u_2 + u_3\}$ is a linearly independent set of vectors. □

3.1 # 11 Suppose that a linear transformation T satisfies

$$T(u_1) = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \quad T(u_2) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad T(u_3) = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

Find $T(-u_1 + 4u_2 - 3u_3)$

Solution Recall a linear transformation $T : U \to V$ satisfies for $a, b \in \mathbb{R}$ and $x, y \in U$

$$T(ax + by) = aT(x) + bT(y)$$

In our case we see

$$T(-u_1 + 4u_2 - 3u_3) = -T(u_1) + 4T(u_2) - 3T(u_3) = -\begin{pmatrix} -3 \\ 0 \end{pmatrix} + 4\begin{pmatrix} 2 \\ -1 \end{pmatrix} - 3\begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} 3+8 \\ -4-15 \end{pmatrix} = \begin{pmatrix} 11 \\ -19 \end{pmatrix}$$

□