Tutorial #1

MAT 188 – Linear Algebra I – Fall 2015

SOLUTIONS

Problems (Please note these are from Holt's Linear Algebra Text on pg.13)

1.1 #52: True or False A linear system with three equations and five variables must be consistent?

Solution This is false. Consider the counterexample of

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 6 \\ x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 6 \\ x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 0 \end{aligned}$$

Clearly the bottom equation violates the other two, which means there is no solution to the system (i.e. it's inconsistent). □

1.1 #54: True or False A triangular system always has exactly one solution?

Solution This is true. We know the system is given by (choosing a upper triangle system without the loss of generality, $a_{ii} \neq 0$ for $i \in [1, n]$)

$$\begin{array}{ccccccccccc} a_{11}x_1 & + & a_{12}x_2 & + & a_{13}x_3 & + & \ldots & + & a_{1n}x_n & = & b_1 \\ & & a_{22}x_2 & + & a_{23}x_3 & + & \ldots & + & a_{2n}x_n & = & b_2 \\ & & & & a_{33}x_3 & + & \ldots & + & a_{3n}x_n & = & b_3 \\ & & & & & \ddots & & & \vdots & & \vdots \\ & & & & & & \ddots & & & & \\ & & & & & & & & a_{nn}x_n & = & b_n \end{array}$$

Thus we may read off the last equation to deduce

$$x_n = \frac{b_n}{a_{nn}}$$

If we write down the second last equation, we see that

$$a_{n-1,n-1}x_{n-1} + a_{n-1,n}x_n = b_{n-1} \implies x_{n-1} = \frac{b_{n-1} - a_{n-1,n}x_n}{a_{n-1,n-1}} = \frac{a_{n,n}b_{n-1} - a_{n-1,n}b_n}{a_{nn}a_{n-1,n-1}}$$

We see a pattern! We'll always be able to solve x_i if we know x_j with $j \in [i+1, n]$. Since we have explicit formulas for every x_i, we see a triangular system has exactly one solution. □

1.1 #63: Calculation A total of 385 people attend the premiere of a new movie. Ticket prices are \$11 for adults and \$8 for children. If the total revenue is \$3974, how many adults and children attended?

Solution First build a system. The two natural variables are adults(A) and children(C). Since we know 385 people attended the movie, we know that

$$A + C = 385$$

The other piece of information was adult tickets cost \$11 and child tickets cost \$8 and the total revenue was \$3974. This means that

$$11A + 8C = 3974$$

Now let's try to solve the system we've created. From the first equation we see that $A = 385 - C$, plug this into the second equation to isolate C,

$$11A + 8C = 3974 \implies 11(385 - C) + 8C = 3974 \implies 4235 - 3C = 3974 \implies 261 = 3C \implies C = 87$$

The equation we substituted gives us the number of adults now

$$A = 385 - C = 385 - 87 = 298$$

Thus the number of children and adults that attended the movie were 87 and 298 respectively. □