DUNTROON WORKSHOP

1. Monday. Arithmetic motivation.

1.1. **Statement of Fundamental lemma.** Geometric interpretation.
Refs: [FL, intro, 1.7, 1.11, 1.13], [D-N, intro, 2.1-2.2], [D4], [Harris]

1.2. **Role of Fundamental Lemma in trace formula.** Refs: [Harris]

2. Tuesday. Lie theory and bundles on curves.

2.1. **Adjoint quotient, characteristic polynomial, Kostant slice, regular centralizers.** Semisimple Lie groups, Borel subgroups, maximal tori. Fundamental diagram

$$\tilde{\mathfrak{g}}/G \xrightarrow{\mu} \mathfrak{g}/G \underset{\chi}{\overset{\epsilon}{\rightleftarrows}} \mathfrak{h}/\!/W.$$

and its $\mathbb{G}_m$-equivariant properties. Rough structure of fibers of χ, μ. Regular centralizers: group scheme $\mathcal{J} \to \mathfrak{h}/\!/W$ of automorphisms of image of ϵ, group stack $B\mathcal{J} \to \mathfrak{h}/\!/W$. Emphasize example of $G = GL_n$ throughout.
Refs: [FL, 1.1-1.2, 2.1-2.3], [D-N, 1.2-1.3], [lect, 3], [D1, D3]

2.2. **Higgs fields, Hitchin fibration, symmetries of Hitchin fibration.** Maps into fundamental diagram and $\mathbb{G}_m$-equivariant variant. Maps into Picard stack $B\mathcal{J} \to \mathfrak{h}/\!/W$. Emphasize domains: complete smooth curve C, formal disk $D = \operatorname{Spec} k[[t]]$, formal punctured disk $D^\times = \operatorname{Spec} k((t))$.
Refs: [FL, 4.1-4.3], [D-N, 3.1-3.4], [lect, 2, 4], [BD]

2.3. **Language of spectral curves.** Interpret preceding two lectures in terms of spectral (and possibly cameral) curves. Examples of Hitchin fibers and Picard symmetries for various spectral curves. Regular locus.
Refs: [FL, 4.4-4.5], [Don], [lect, 5], [D2]

2.4. **Distinguished subsets of Hitchin base.** Geometry of discrimant locus in $\mathfrak{h}/\!/W$. Generically regular spectral curves. Smooth spectral curves. Invariant δ.
Refs: [FL, 1.10, 4.6-4.9], [D-N, 3.7], [D2]

2.5. **Stratification of Hitchin base.** More on the invariant δ. Normalization of spectral curves.
Refs: [FL, 5], [D-N 3.9]

3. Wednesday. Global vs local geometry of bundles.

3.1. **Uniformization of bundles, affine Grassmannian.** Gluing of bundles. Bundles with trivialization. Geometry of affine Grassmannian: symmetries, Schubert stratification.
Refs: [Sor], [BD]

3.2. **Affine Springer fibers, general properties and symmetries.** Affine Springer fibers via analogy with ordinary Springer fibers. Affine Springer fibers as local Higgs fields. Precise dependence on defining data. Structure of group of symmetries, in particular its components. General properties: projectivity, dimension.
Refs: [FL, 3.2-3.4, 3.7], [D-N, 2.3]

3.3. **Examples of affine Springer fibers, regular characteristic polynomials.** Examples of affine Springer fibers for SL_2. Description of affine Springer fibers for smooth spectral curves.
Refs: [GKM]

3.4. **Factorization of affine Grassmannian, Hitchin fibration.** Factorization of generically trivial objects. Relation of global Hitchin fibers to affine Springer fibers.

Refs: [FL, 4.15-4.16], [D-N, 3.5]

3.5. **Explanation of problem: Decomposition Theorem and characters applied to Hitchin fibration.** Review of intersection cohomology and Decomposition Theorem.

Pushforward of constant sheaf along Hitchin fibration. Action of coweight lattice on pushforward and decomposition according to characters. Interpretation in terms of local systems. Relation to cohomology of affine Springer fibers.

Refs: [FL, 6.4], [D-N, 2.4-2.5, 5.1-5.2]

4. Thursday. Cohomology of Hitchin fibration.

4.1. **Langlands duality and endoscopic groups from a geometric perspective.** Combinatorics of Langlands duality: based root data, dual groups. Geometry of endoscopic groups: characters, Hitchin bases.

Refs: [FL, 1.8-1.9, 6.3], [D-N, 1.4-1.6, 5.3]

4.2. **Abelian fibrations and Ngo's support theorem.** Statement of support theorem. Example: families of curves. Idea of proof.

Refs: [FL, 7], [AbFib], [D-N, 5.5]

4.3. **Cohomology of Hitchin fibration and affine Springer fibers.** Locus of δ-regularity. Complete proof of Fundamental Lemma.

Refs: [FL, 7.8], [D-N, 5.6-5.8], [AbFib, 4]

4.4. **Examples.**

4.5. **Interpretation in terms of counting points.** Lefschetz fixed point theorem. Counting points of affine Springer fibers.

Refs: [FL, 8], [D-N, 4]

5. Friday. Further directions: traces in geometry.

6. References

[FL] Ngo, Fundamental Lemma:
`http://www.math.ias.edu/~ngo/LF.pdf`

[D-N] Dat-Ngo, survey of Ngo's proof:
`http://www.math.jussieu.fr/~dat/recherche/publis/proj_livre.pdf`

[AbFib] Ngo, on abelian fibrations:
`http://www.institut.math.jussieu.fr/projets/fa/bpFiles/Ngo.pdf`

[ICM] Ngo, Madrid ICM talk:
`http://www.math.u-psud.fr/~ngo/ICM-NgoBaoChau.pdf`

[lect] Ngo, lecture notes:
`http://www.staff.science.uu.nl/~looij101/hitchinfibration.pdf`

Drinfeld's notes:

[D1] http://www.math.uchicago.edu/%7Emitya/langlands/Kostant-Theorem.pdf
[D2] http://www.math.uchicago.edu/~mitya/langlands/Hitchin_fibration.pdf
[D3] http://www.math.uchicago.edu/~mitya/langlands/Regular_centralizers.pdf
[D3] http://www.math.uchicago.edu/%7Emitya/langlands/SL-n.pdf

[BD] Beilinson-Drinfeld, Quantization preprint:
http://www.math.uchicago.edu/%7Emitya/langlands/hitchin/h1-100.ps
http://www.math.uchicago.edu/%7Emitya/langlands/hitchin/h101-200.ps
http://www.math.uchicago.edu/%7Emitya/langlands/hitchin/h101-200.ps
http://www.math.uchicago.edu/%7Emitya/langlands/hitchin/h301-384.ps

[Don] Donagi, spectral curves:
http://arxiv.org/abs/alg-geom/9505009

[Sor] Sorger, lectures on G-bundles:
http://users.ictp.it/~pub_off/lectures/lns001/Sorger/Sorger.pdf

[GKM] Goresky-Kottwitz-MacPherson, on affine Springer fibers:
http://arxiv.org/abs/math/0305144

[Harris] Paris book project:
http://fa.institut.math.jussieu.fr/node/44