Math 327, Term Test
Fall Quarter 2011
Thu, Oct 27

Name: ________________________

Instructions:
Show *all your work* on these sheets. Justify your answers! This test has 5 problems and 7 pages. Make sure you have all of them. No calculators, books, notes, etc. are allowed.

Prob.	Possible points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
TOTAL	50	

Question 1. *(10 points)*

(a) Define the concept of a basis for a topology on a set X.

(b) Is the collection of all closed intervals, $\mathcal{C} = \{[a, b] : a < b \text{ in } \mathbb{R}\}$, a basis for a topology on the set of real numbers?

(c) Suppose that X, Y are topological spaces. Show that $\{U \times V : U \text{ open in } X, V \text{ open in } Y\}$ is a basis for a topology on $X \times Y$. ***You are not allowed to assume anything about the product topology!***

Question 2. *(10 points)*

(a) Show that $\mathbb{R}_\ell$ *is disconnected.*

(b) Show that if X *is path-connected, then* X *is connected.*

(c) Is $\mathbb{R}_\ell$ *path-connected?*

Question 3. *(10 points) Suppose that* $A \subset X$ *and suppose that* $f : A \to Y$ *is a continuous function with* Y *Hausdorff. Show that there is at most one continuous function* $g : \overline{A} \to Y$ *that extends* f *(that is,* $g|_A = f$*).*

Question 4. *(10 points) Recall that, as a set,* $\mathbb{R}^\omega = \prod_{n=1}^{\infty} \mathbb{R}$.

(a) Is the function $f : \mathbb{R} \to \mathbb{R}^\omega$ *with* $f(t) = (t, 2t, 3t, \dots)$ *continuous, if* $\mathbb{R}^\omega$ *carries the product topology?*

(b) Does the sequence in $\mathbb{R}^\omega$*,*

$$\begin{aligned} \mathbf{x}_1 =&(1, 0, 0, 0, \dots) \\ \mathbf{x}_2 =&(\frac{1}{2}, \frac{1}{2}, 0, 0, \dots) \\ \mathbf{x}_3 =&(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, \dots) \\ &\vdots \end{aligned}$$

converge, if $\mathbb{R}^\omega$ *carries the box topology? Justify your answer.*

Question 5. *(10 points) Suppose that* (X, d) *is a metric space. Fix any element* $x \in X$ *and* $r > 0$*, a positive real number.*

(a) Show that $C_r(x) = \{y \in X : d(x, y) \leq r\}$ *is a closed subset of* X*.*

(b) Give an example to show that $\overline{B_r(x)}$ *need not be equal to* $C_r(x)$*. (Recall that* $B_r(x) = \{y \in X : d(x, y) < r\}$*. Hint: one can find such an example that is a subset of* $\mathbb{R}$ *with the induced metric.)*

Scratch Paper