My ring theory conventions, MAT347

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I will list in this note any differences in my conventions from those of our textbook, Dummit&Foote.

- A *ring* always contains a multiplicative identity 1.
- A *subring* of R has to contain the multiplicative identity of R.
- A *ring homomorphism* has to send 1 to 1.
- A *ideal* is a subgroup I of $(R, +)$ such that $rI \subset I$ and $Ir \subset I$ for all $r \in R$ (for short, $RI \subset I$ and $IR \subset I$). Similarly for left/right ideals.

Examples:

- For us, $2\mathbb{Z}$ is *not* a ring, and $\mathbb{Z} \times 0$ is *not* a subring of $\mathbb{Z} \times \mathbb{Z}$ (since it doesn't contain the identity $(1, 1)$).
- For us, the only "trivial ring" (Dummit&Foote, p. 224) is the zero ring 0.
- For us, the only (left/right/2-sided) ideal of R that is a subring is R itself! (Reason: any subring contains 1, if an ideal contains 1 it has to be the whole ring.)
- For us, every ring $R \neq 0$ contains a maximal ideal, and more generally every proper ideal is contained in a maximal ideal. (Compare with Prop. 11 in Sec. 7.4.)
- For us, if $\phi : R \to S$ is a homomorphism of commutative rings and P is a prime ideal of S, then $\phi^{-1}(P)$ is a prime ideal of R. (Compare with Ex. 13(a) in Sec. 7.4.)