MAT 347
Problems for Homework 19
March 19, 2020

1. Let $a \in \mathbb{F}_p$ and consider $f(x) := x^p - x - a \in \mathbb{F}_p[x]$. Let K be the splitting field of $f(x)$ over $\mathbb{F}_p$. **Update (April 4):** please assume $a \neq 0$, otherwise f is reducible.

   (a) Show that f is separable and deduce that $K/\mathbb{F}_p$ is a Galois extension.

   (b) Show that there is an $\alpha \in K$ such that $f(x) = \prod_{i=0}^{p-1}(x - \alpha - i)$. (Hint: show that if α is a root, then so is...) Deduce that $K = \mathbb{F}_p(\alpha)$.

   (c) Show that Frobenius is in $\mathrm{Gal}(K/\mathbb{F}_p)$. What is its action on the set of roots of f?

   (d) Compute the order of Frobenius in the Galois group and deduce that $f(x)$ is irreducible over $\mathbb{F}_p$.