MAT347 – Groups, Rings, and Fields, 2018–2019

Logistics

- **Instructor:** Florian Herzig (herzig@math.toronto.edu)
  Office: BA 6186
  Office Hours: see course website.
- **TA:** Daniel Spivak (daniel.spivak@mail.utoronto.ca)
- **Textbook:** Dummit and Foote, *Abstract algebra*, Wiley. 3rd edition.
- **Course Webpage:** The website for this course is

  http://www.math.toronto.edu/~herzig/347-f18.html.

  Please check this site regularly to see what we are covering each week, and to find assignments. Quercus will mainly be used for posting grades.
- **Lectures:**
  Mon 3–4 (GB 221) and Wed 3–5 (GB 221)
- **Tutorials:**
  Tue 10–11 and Tue 11–12, depending on which you have been assigned to. Tutorials meet in ES B149 in the Fall and BA 1240 in the Winter. Tutorials are an important part of the course!
- **Drop deadline:** February 18, 2019.
- **Prerequisites:** The calendar lists MAT 257 (and hence indirectly MAT157, MAT240, and MAT247) as prerequisites, and I will assume you have taken these courses. Specifically, in terms on content, you need to be perfectly comfortable with abstract linear algebra (that is, over arbitrary fields, including finite fields) and with complex numbers. However, more important than having learned any piece of content, the prerequisites are there to guarantee that you are comfortable with abstract reasoning and proof techniques, and that you understand how to write and read mathematics. If you are uncertain of whether you have the necessary prerequisites, talk to me.
- **Accessibility Needs:** If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom or course materials, please contact Accessibility Services as soon as possible:

  http://www.studentlife.utoronto.ca/as.

Content objectives of the course

This course will provide a detailed introduction to modern abstract algebra, which is a basic part of the language of much of modern mathematics. Consider the following questions:

- You probably know how to construct a regular hexagon with straight edge and compass, but for which values of n is a regular n-gon constructible? For instance, a regular heptagon is impossible to construct, but a regular 17-gon is constructible.
- You are familiar with the quadratic formula that allows us to solve the equation $ax^2+bx+c=0$ using radicals. There is a similar formula for cubic and quartic equations, but it is impossible to find such a formula for quintic equations. Even more, it is impossible to write down the solutions to the equation $x^5 - 6x + 3 = 0$ explicitly in terms of the four basic operations, rational numbers, and n-th roots. Why?
- You may have heard that the function $F(x) = \int_0^x e^{t^2} dt$ is transcendental. In other words, we cannot write an anti-derivative of $f(x) = e^{x^2}$ in terms of "elementary functions" (i.e. as composition of rational functions, radicals, trig functions, exponentials, and logarithms). You also may have heard that the differential equation $y'' + xy = 0$ cannot be solved in terms of elementary functions and anti-derivatives. How do we prove these statements?

The answers to all these problems lie in Galois theory, one of the most beautiful gems of mathematics that you will learn as an undergraduate (which Galois developed at the age of 17, four years before dying in a duel), and the ultimate goal of this course. To get there, first we need to develop a lot of background: group theory (first term), and rings and fields (second term). By the end of the course we will have solved problems 1 and 2 and you will have an idea of how problem 3 could be attacked. The path is long, but the reward at the end of the way is well worth it.

Course structure

- **Book chapters.** We will mostly cover material from Chapters 0–5, 7–9, 13–14. Occasionally I may introduce some material not in the book. If this is significant, I will state so.
- **Classroom time.** All four hours of class time are equally important. Do not think of them as three hours of lecture plus one hour of tutorial.

One of the objectives of the course is to continue your transformation from students to mathematicians. You will not accomplish this by merely watching me do mathematics, but by doing lots of math yourselves. Hence there will be important parts of the course, including big theorems, that you will be responsible for proving yourself. This includes some class time.

The textbook has more space than we have class meeting time. I prefer to spend class time motivating difficult concepts or elaborating on harder examples or guiding you to construct your own proofs rather than writing the details of an easy proof that is found in the book and that you could reconstruct on your own.

- **Reading.** There will be roughly weekly reading assignments from the textbook, corresponding to the material that we are covering. Even if we do not complete all details in class, I still expect you to read the entirety of each chapter and to understand it, in addition to class notes. In other words, everything is equally important, and everything prepares you equally well for the tests.

- **Homework:**
  There will be roughly weekly homework assignments that I ask you to complete. The assignments are long, but you are only required to turn in a small number of problems each week, which will be marked. I choose problems that will help solidify the concepts and that will help you understand the subsequent lectures. Because of that, it is important that you do *all* of the assigned problems, including the ones not to be turned it, and I will assume you have. This is particularly relevant for exams. **No late assignments will be accepted.** This course is unforgiving if you fall behind, so I do not want to encourage it. The lowest three scores will be dropped for every student. We will post the homework assignments on the course website and grades on Quercus.

- **Term Tests:**
  There will be two term tests, taking place tentatively on Wednesday, November 14, 3–5pm and Wednesday, February 6, 3–5pm, during class time. More details about the test will be provided on the course website, as the time approaches.

  Note: There will be **no** makeup term test! If you miss the term term and you do not provide a valid reason (e.g., a doctor's note) within one week of the test, the test grade will be counted as 0. If you provide a valid reason within one week, the marking scheme will be adjusted appropriately.

- **Final Exam:**
  The final exam will take place during the examination period in April 2019,

and will be three hours long. It will cover all the material presented in lectures and tutorials. No aids will be allowed. More details about the final will be provided as the time approaches.

- **Marking Scheme:**
  Your final mark will consist of homework (10%), two midterms (25% each), and final exam (40%).

- **A word of caution.** Even though it may seem like we start slowly, we will pick up the pace, and I guarantee you you will be doomed if you fall behind. That would be a pity as the final part of this course is pure beauty.

- **Code of Behaviour / Plagiarism:**
  Students should become familiar with and are expected to adhere to the Code of Behaviour on Academic Matters which can be found at

  `http://www.governingcouncil.utoronto.ca/policies/behaveac.htm`.