MAT 347
Order
September 10, 2019

Order

Let G be a group. Let $a \in G$. We want to compare the powers of a. In other words, when do we have $a^n = a^m$?

First, we define the *order* of a as the smallest positive integer n such that $a^n = 1$, if there is such a thing. Otherwise we define the *order* of a to be infinity. We denote the order of a by $|a|$.

1. Let G be a group, let $a \in G$, and let $r = |a|$. Complete the following statements and prove them:

   (a) $a^n = 1 \iff \ldots$ (something about n and r)

   (b) $a^n = a^m \iff \ldots$ (something about n, m, and r)

2. Find the order of every element in $\mathbb{Z}/12\mathbb{Z}$ (under addition) and $(\mathbb{Z}/18\mathbb{Z})^\times$ (under multiplication).
   Note: This questions is shorter than it seems.

3. Find an example of a group G that contains one element of order n for every positive integer n and which also contains an element of order infinity.

Order in symmetric groups

The cycle notation for symmetric groups is well-adapted to finding the order of elements.

4. What is the order of a k-cycle?

5. What is the order of the following elements of S_6?

   (a) (12)(34)(56)

   (b) (12)(345)

   (c) (123)(456)

6. Given a permutation expressed as a product of disjoint cycles, explain how you would compute its order.

7. What is the maximal order of an element in S_7?