MAT 347
Orbits and Stabilizers
September 25, 2018

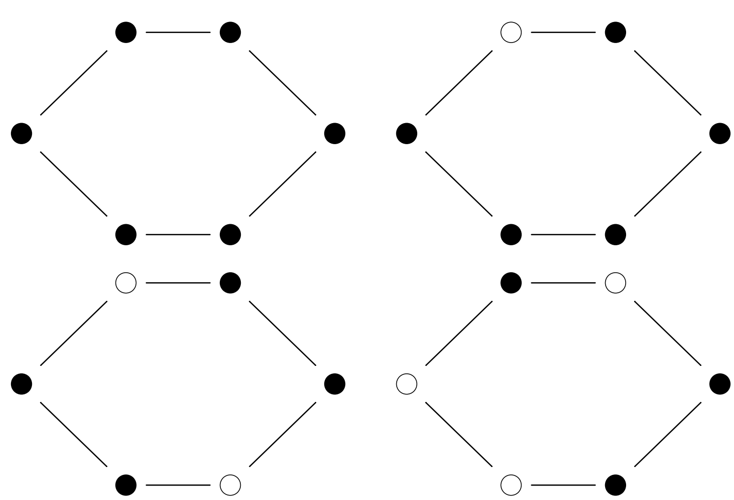
Orbits and Stabilizers

Definitions. Let G be a group acting on a set A.

- Given $g \in G$, we define the *fixed set* of g as the set
$$\text{Fix}(g) := \{a \in A \mid g \cdot a = a\} \subseteq A.$$
- Given $a \in A$, we define the *stabilizer* of a as the set
$$\text{Stab}(a) := \{g \in G \mid g \cdot a = a\} \subseteq G.$$
- Given $a \in A$ we define the *orbit* of a as the set
$$Ga := \{g \cdot a \mid g \in G\} \subseteq A.$$

1. Verify that $\text{Stab}(a)$ is a subgroup of G.
2. Say we want to count how many *different* necklaces we can build with 6 stones each, if we have stones of two different colours. Define a *diagram* to be any way of colouring each of the six vertices of a hexagon with black or white. Notice that $|A| = 64$. Show that D_{12} acts on A, and that the number of orbits of this action equals the number of different necklaces.

   Note: This shows that the problem of counting the number of orbits of an action is an interesting problem in combinatorics.
3. Regarding the previous question, consider the following diagrams:



For each one of them, compute the size of its orbit and the size of its stabilizer. Make a conjecture or a formula that relates these two numbers for an arbitrary element in an arbitrary action. Then prove it.

4. If a group G acts on a set A and Ga, Gb are any two orbits, what can we say about how Ga and Gb relate to each other? (For example, what happens if the two orbits have any element in common?)