Presentations

A presentation of a group is a set S of generators along with relations $R_1, \ldots, R_m$, which are equations in the generators. The resulting group $G = \langle S \mid R_1, \ldots, R_m \rangle$ consists of words $xyz^{-1}xy^{-1}zz\cdots$ in the generators (where $S = \{x, y, z, \ldots\}$), except that two words are equal if we can simplify them using the relations.

1. For each of the following presentations, figure out how many elements are the resulting group and then try to recognize the group.

    (a) $\langle a \mid a^n \rangle$, where $n \geq 1$ is fixed.

    (b) $\langle s, t \mid s^2 = t^2 = 1, sts = tst \rangle$.

    (c) $\langle a, b \mid ba = ab^2, ab = ba^2 \rangle$.

    (d) $\langle a, b \mid a^4 = 1, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$.

2. If a group is presented with only one generator, what can you say about the group?