# MAT 1200/415, Algebraic Number Theory, Fall 2018
## Homework 1, due on Friday September 28
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1. Marcus, Number fields, Chapter 2, Problems 11, 14, 30, 42(ab).

2. Suppose that $A \subset B$ are domains with $B$ integral over $A$, and that $\mathfrak{q}$ is a prime ideal of $B$.

   (a) Show that $A$ is a field iff $B$ is a field.

   (b) Deduce that $\mathfrak{q}$ is maximal in $B$ iff $\mathfrak{q} \cap A$ is maximal in $A$.

3. Let $K = \mathbb{Q}(\sqrt{-14})$. Let $I = (3, \sqrt{-14} - 1)$ be an ideal in $\mathcal{O}_K$. Prove that $I$, $I^2$, $I^3$ aren't principal, while $I^4$ is.

4. Let $I$ be the ideal $(2, 1 + \sqrt{-3})$ in $\mathbb{Z}[\sqrt{-3}]$. Prove that $I^2 = 2I$ but $I \neq (2)$. Conclude that ideals in $\mathbb{Z}[\sqrt{-3}]$ do not factor uniquely into prime ideals. (We'll soon see that in rings of integers of number fields we do have unique factorisation into prime ideals. Why isn't this a contradiction?)