FUNDAMENTAL CONCEPTS IN DIFFERENTIAL GEOMETRY
FALL 2000
EXERCISES HANDOUT # 12

1. (a) Show that a rank n vector bundle is trivial, if and only if there exists n linearly independent sections (i.e. $s_1, \dots, s_n$ such that for any b in the base space $s_i(b)$ are linearly independent.)

(b) Show that the Moebius band is the total space of a non trivial vector bundle of rank 1 over the circle.

(c) Show that $Mb \oplus Mb$ is a trivial bundle over the circle.

2. Define a C^∞ structure of a manifold on TM in such a manner that for each coordinate system (U, φ) on M, with local coordinates $(x^1, \cdots, x^n)$ and frames $E_1, \dots, E_n$, the set $\tilde{U} = \pi^{-1}(U)$ with mapping $\tilde{\varphi}: \tilde{U} \to R^{2n}$ defined as follows is a coordinate neighborhood. For $p \in U$ and $X_p \in \tilde{U}$, we suppose that $X_p = \sum \alpha^i E_{ip}$ and define

$$\tilde{\varphi}(X_p) = (x^1(p), \dots, x^n(p), \alpha^1, \dots, \alpha^n).$$

3. A manifold M is called parallelizable, if TM is a trivial bundle.

(a) Show that S^3 is parallelizable.

(b) Is the Klein bottle parallelizable ?

(c) Is S^2 parallelizable ?

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