

**Our Algebras.** Let  $sl_{2+}^\epsilon := L\langle y, b, a, x \rangle$  subject to  $[a, x] = x$ ,  $[b, y] = -\epsilon y$ ,  $[a, b] = 0$ ,  $[a, y] = -y$ ,  $[b, x] = \epsilon x$ , and  $[x, y] = \epsilon a + b$ . So  $t := \epsilon a - b$  is central and if  $\exists \epsilon^{-1}$ ,  $sl_{2+}^\epsilon / \langle t \rangle \cong sl_2$ .

Indeed if  $\epsilon$  is invertible, the map

$$\phi_\epsilon: sl_{2+}^\epsilon \rightarrow sl_{2+}^1, \quad (y, b, a, x) \mapsto (\epsilon y, \epsilon b, a, x),$$

is an isomorphism of Lie algebras, and  $sl_{2+}^1 / \langle t \rangle \cong sl_{2+}^1 / (a = b) \cong L\langle y, a, x \rangle / ([a, x] = x, [a, y] = -y, [x, y] = 2a) \cong sl_2$ .

$U$  is either  $CU = \mathcal{U}(sl_{2+}^\epsilon)[[\hbar]]$  or  $QU = \mathcal{U}_\hbar(sl_{2+}^\epsilon) = A\langle y, b, a, x \rangle[[\hbar]]$  with  $[a, x] = x$ ,  $[b, y] = -\epsilon y$ ,  $[a, b] = 0$ ,  $[a, y] = -y$ ,  $[b, x] = \epsilon x$ , and  $xy - qyx = (1 - AB)/\hbar$ , where  $q = e^{\hbar\epsilon}$ ,  $A = e^{-\hbar\epsilon a}$ , and  $B = e^{-\hbar b}$ . Set also  $T = A^{-1}B = e^{\hbar t}$ .

**The Quantum Leap.** Also decree that in  $QU$ ,

$$\Delta(y, b, a, x) = (y_1 + B_1 y_2, b_1 + b_2, a_1 + a_2, x_1 + A_1 x_2),$$

$$S(y, b, a, x) = (-B^{-1}y, -b, -a, -A^{-1}x),$$

and  $R = \sum \hbar^{j+k} y^j b^k \otimes a^j x^k / j! [k]_q! = 1 + \hbar r + O(\hbar^2)$ , where  $r = y \otimes x + b \otimes a$  satisfies CYBE,  $[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0$ .  $\phi_\epsilon$  extends to  $QU$  with  $\phi_\epsilon(\hbar) = \hbar/\epsilon$  and  $R$  is  $\phi_\epsilon$ -invariant.

**The  $s, t$  Alternative.** With  $s = \epsilon a + b$  (and  $t = \epsilon a - b$ ), we have that  $sl_{2+}^\epsilon$  is

$$L\langle t, y, s, x \rangle / ([t, \cdot] = 0, [s, x] = 2\epsilon x, [s, y] = -2\epsilon y, [x, y] = s),$$

with  $r = y \otimes x + (s-t) \otimes (s+t)/4\epsilon \sim y \otimes x + (s \otimes s + s \otimes t - t \otimes s)/4\epsilon$ .

**The  $d, t$  Alternative.** With  $d = s/\epsilon$ , this becomes

$$L\langle t, y, d, x \rangle / ([t, \cdot] = 0, [d, x] = 2x, [d, y] = -2y, [x, y] = \epsilon d),$$

with  $r \sim y \otimes x + (\epsilon d \otimes d + d \otimes t - t \otimes d)/4$ .