From Stonehenge to Drinfel'd Skipping all the Details

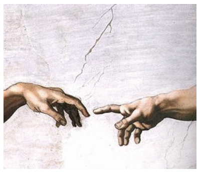
Stonehenge



Drinfel'd

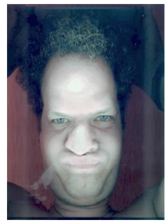
Lehigh University Geometry/Topology Conference, June 11−13, 2000

Dror Bar−Natan, Hebrew University and MSRI

Creation of Adam

Michelangelo

Disclaimer

1. We'll concentrate on the beauty and ignore the cracks.
2. The speaker is an idiot.

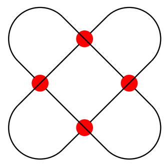

picture taken by a flatbed scanner, November 1999.

The Gaussian linking number

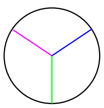
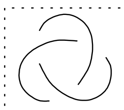
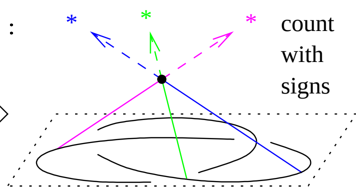
$lk(\bigcirc\!\!\bigcirc) = \frac{1}{2}\sum$ (signs)

vertical chopsticks


Carl Friedrich Gauss


→ $lk=2$

$\langle D,K\rangle_{\text{Stonehenge}} :=$ (The signed Stonehenge pairing of D and K) :

$D=$

$K=$

⇒

count with signs

The generating function of all stellar coincidences:

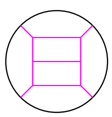
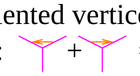
$$Z(K) := \lim_{N\to\infty} \sum_{3\text{-valent } D} \frac{1}{2^c c! \binom{N}{e}} \langle D,K\rangle_{\text{Stonehenge}} D \cdot \left(\begin{matrix}\text{framing-}\\ \text{dependent}\\ \text{counter-term}\end{matrix}\right) \in \mathcal{A}(\circlearrowleft)$$

with

N := # of stars

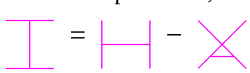
c := # of chopsticks

e := # of edges of D

$\mathcal{A}(\circlearrowleft)$:=Span $\left\langle\right.$  $\left.\right\rangle$ / oriented vertices, AS:  =0, & more relations

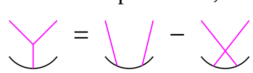
When deforming, catastrophes occur when:

A plane moves over an intersection point − Solution: Impose IHX,



(see other side)

An intersection line cuts through the knot − Solution: Impose STU,

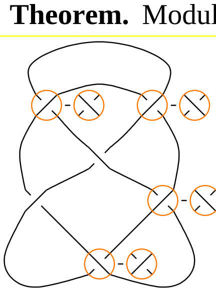


(similar argument)

The Gauss curve slides over a star − Solution: Multiply by a framing−dependent counter−term.

(not shown here)

Theorem. Modulo Relations, *Z(K)* is a knot invariant!


The Miller Institute knot

Definition. *V* is finite type (Vassiliev) if it vanishes on sufficiently large alternations as on the left.

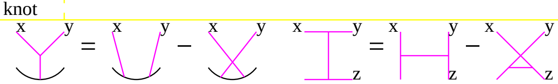
Theorem. All knot polynomials (Conway, Jones, etc.) are of finite type.

Conjecture. (Taylor's theorem) Finite type invariants separate knots.

Theorem. *Z(K)* is a universal finite type invariant!

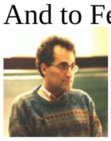
(sketch: to dance in many parties, you need many feet).

Related to Lie algebras




Sophus Lie

And to Feynmann diagrams for the Chern−Simons−Witten theory:

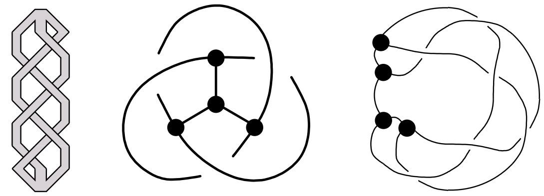

Edward Witten

$$\int_{\mathfrak{g}\text{-connections}} \mathcal{D}A\, hol_K(A) \exp\left[\frac{ik}{4\pi}\int_{\mathbb{R}^3} \text{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)\right]$$

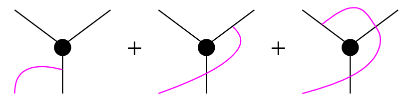
Computing *Z(K)*:

☹ "Crossing change" is not well defined!

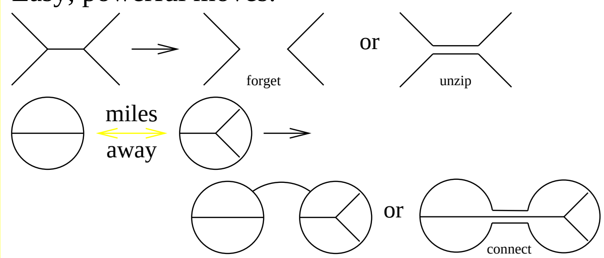
☺ Switch to Embedded Trivalent (ribbon) Graphs:

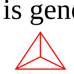


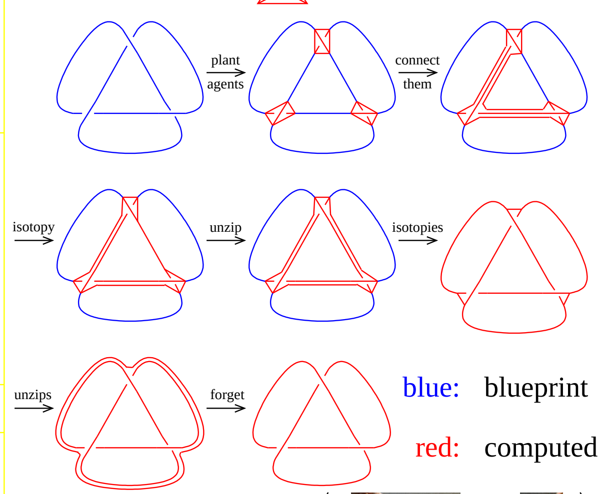
Need a new relation:



Easy, powerful moves:

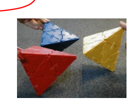
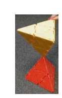


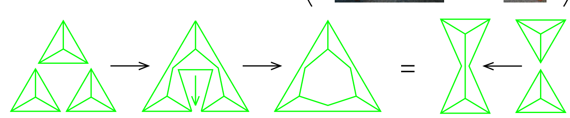
Using moves, ETG is generated by ribbon twists and the tetrahedron  :



blue: blueprint

red: computed

Modulo the relation(s): ( = )



(+more)

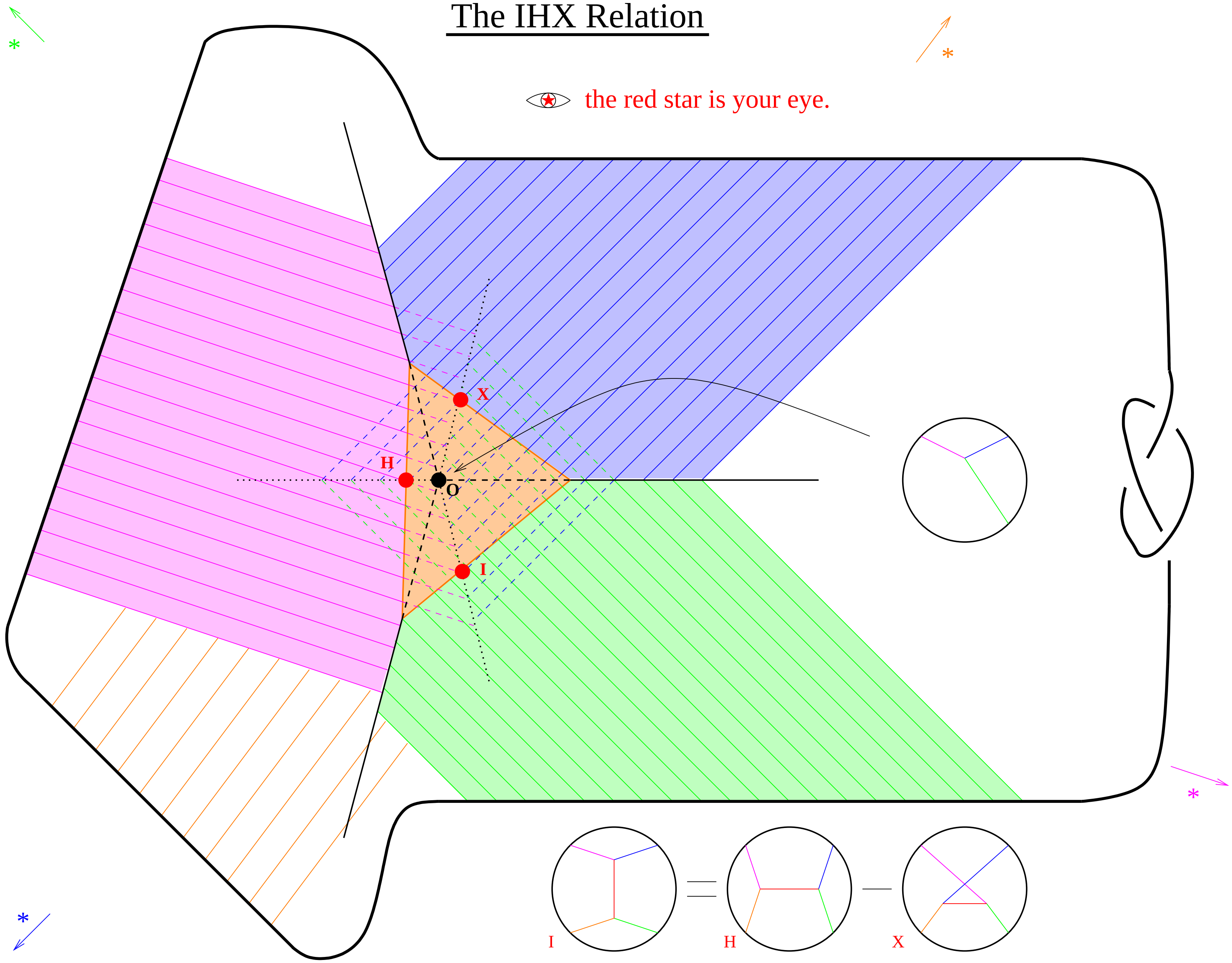
Claim. With $\Phi := Z(\text{tetrahedron})$, the above relation becomes equivalent to Drinfel'd's pentagon equation of the theory of quasi−Hopf algebras:

$$(11\Delta)(\Phi)\cdot(\Delta 11)(\Phi) = (1\Phi)\cdot(1\Delta 1)(\Phi)\cdot(\Phi 1)$$

This handout is at http://www.ma.huji.ac.il/~drorbn/Talks/Lehigh−0006

The IHX Relation

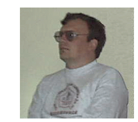
the red star is your eye.



The Cast
(in approximate historical order)

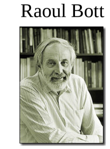
The Neolithic People

Carl Friedrich Gauss
Sophus Lie
Edward Witten
Mikhail Nikolaevich Goussarov



Victor Vassiliev
Maxim Kontsevich

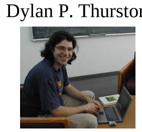
Raoul Bott



Clifford Henry Taubes

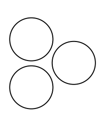
Thang T. Q. Le
Jun Murakami

Tomotada Ohtsuki
Dylan P. Thurston

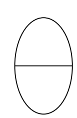


To keep awake:

1. Is there a non−trivial embedding of 3 circles so that if any one of them is dropped the remaining two are unlinked?



2. Is there a non−trivial embedding of a ribbon theta graph so that if any edge is dropped the remaining circle is unknotted?



3. Is there a non−trivial embedding of the skeleton of a tetrahedron so that if any edge is dropped, the remaining theta graph is trivially embedded?