Statistics 2211 Assignment 2

Due Thursday, February 23 beginning of class

First four problems – more problems to come

Problem 1. Let $F : \mathbb{R} \to \mathbb{R}$ be Lipschitz-L, that is $|F(x) - F(y)| < L|x - y|$ for all x, y. Show that $\text{Var}F(X) \leq L^2\text{Var}X$ for all finite mean random variables X.

Problem 2. Let $F : \mathbb{R}^n \to \mathbb{R}$ be Lipschitz-L in all coordinates, let $M_n = F(X_1, \ldots, X_n|\mathcal{F}_k)$ where $\mathcal{F}_k = \sigma(X_1, \ldots X_k)$, and the X_i are independent with finite expectation. Show that

$$\mathbf{E}[(M_k - M_{k-1})^2|\mathcal{F}_{k-1}] \leq L^2\text{Var}X_k.$$

Problem 3. Let M_k be a martingale with bounded increments $|M_{k+1} - M_k| \leq b_k$. Show that $Y_k = \exp(M_k - \sum_{i=1}^k b_k^2/2)$ is a supermartingale.

Problem 4. Show that the chromatic number of an ER random graph $G \sim \mathcal{G}(n, p)$ satisfies $P(|\chi(G)) - \mathbf{E}\chi(G)| > \gamma\sqrt{n}) \leq ca^\gamma$ for some universal constants $c > 0, a < 1$.