Statistics 2211 Assignment 1

Due Thursday, January 21 beginning of class

Problem 1. (a) Let $\mathcal{F}$ be a σ-field on a countable set Ω. Show that there exists pairwise disjoint sets $\Omega_i, i \in I$ so that

$$\mathcal{F} = \{ \bigcup_{i \in J} \Omega_i \, : \, J \subset I \}.$$

(b) Give a counterexample to the above in the case Ω is not countable.

Problem 2-9. Exercises 4.1.x for $x = 2, 3, 4, 5, 6, 7, 8, 9$ in Durrett.

Problem 10. Show that if for all $a \in \mathbb{R}$ we have $\mathbf{E}|X - a| = \mathbf{E}|\mathbf{E}(X - a|\mathcal{G})|$ then $X = \mathbf{E}(X|\mathcal{G})$ a.s.