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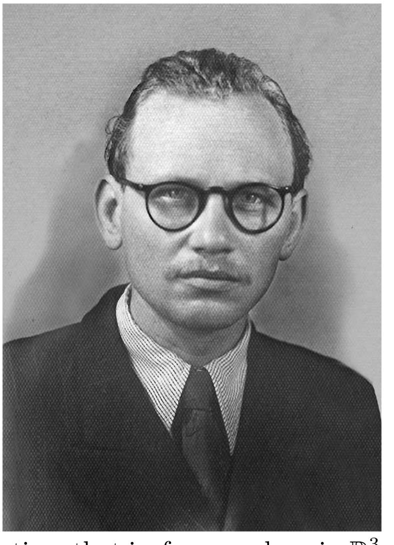
Georgy Maksimovich Adelson-Velsky (obituary)



For most of us he was a friend, a teacher, an elder comrade, a colleague. A talented mathematician, a scientist with an extremely broad range of interests and knowledge, a man whose mind was exceptionally quick and responsive, he was always ready to start working on any problem.

Georgy Maksimovich Adelson-Velsky was born in Samara on January 8, 1922. In 1940 he enrolled in the Faculty of Mechanics and Mathematics at Moscow State University.

As a fourth-year student, in 1944 he generalized S. N. Bernstein's theorem on a smooth function of two variables whose graph has negative curvature. He introduced the following definition: a continuous function of two variables has *generalized non-positive curvature* if no plane cuts off a 'cap' from the graph of the function, that is, for any plane in $\mathbb{R}^3$, all the connected components into which the plane cuts the graph of the function are unbounded.

Theorem [1]. *If a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ has generalized non-positive curvature, then either it grows at infinity at least as fast as a linear function, or its graph is a cylindrical surface, that is, $f(x,y) = g(ax+by)$.*

Continuing these investigations, he and A. S. Kronrod published a cycle of papers [2]–[4] on geometric properties of functions of two variables having limited smoothness. In [4] they solved the following problem posed by J. Hadamard and N. N. Luzin as being of interest: prove the Cauchy–Goursat theorem (on the analyticity of a function of a complex variable having a derivative at each point) using only qualitative properties of the function (and not the Cauchy integral). This work was awarded the Moscow Mathematical Society Prize in 1946. According to Lyusternik,[1] it was the idea behind this cycle of works that gave rise to the geometric theory of functions of two and several variables (the works of A. S. Kronrod, A. G. Vitushkin, and A. N. Kolmogorov). The landmark achievement in this direction was the solution of Hilbert's 13th problem by Kolmogorov and V. I. Arnold.

[1] L. A. Lyusternik, *Review of the works of G. M. Adelson-Velsky.*

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The problem of a continuous decomposition of a Hilbert space into subspaces which are invariant under the action of all the operators in some algebra $\boldsymbol{R}$ was very popular in 1940–1950. The study of the separable case was completed in two monographs by J. Dixmier. The main contributions were made by F. J. Murray, J. von Neumann, A. M. Gleason, and I. E. Segal in the United States, by P. E. Cartier, R. Godement, and A. Weil in France, and (independently: at that time exchange of information was strongly inhibited) by I. M. Gelfand, M. A. Naimark, and Adelson-Velsky in the Soviet Union.

In 1949 Adelson-Velsky defended his Ph.D. thesis "Spectral analysis of the ring of bounded linear operators in a Hilbert space" under the supervision of Gelfand. His official opponents were Kolmogorov and Naimark. In this work he constructed a decomposition of a normed ring with involution into a direct integral (a generalization of the notion of a direct sum) of irreducible rings of bounded linear operators. No assumptions were made about separability. Naimark singled out three parts of the thesis:

1) construction of a decomposition into independent rings;

2) a counterexample which shows that the components obtained are not necessarily irreducible;[2]

3) a construction that 'corrects' the representation to an irreducible one.

A brief version of the first part was published in [6]. A detailed exposition was never prepared for publication and can be found only in the thesis [5], which is not easily accessible.

A Banach mean on a group $\boldsymbol{G}$ is a right-invariant non-negative linear functional L on the space of bounded real functions on $\boldsymbol{G}$ such that $L(1) = 1$. Suppose that for any finite set $E \subset \boldsymbol{G}$ the number of all possible products of at most n (not necessarily distinct) elements in E grows slower than exponentially as $n \to \infty$. It was shown in [7] that if this condition holds, then there exists a Banach mean on the group $\boldsymbol{G}$. This condition does hold for commutative groups, but not for SO(3). There is no Banach mean on SO(3), as follows from the famous example of Hausdorff (showing that on the unit sphere there does not exist a non-trivial SO(3)-invariant finitely additive measure with respect to which all subsets are measurable). In [7] the authors also gave a (hard to verify) criterion, motivated by Hausdorff's example, for the existence of a Banach mean on a group $\boldsymbol{G}$.

In 1955 Adelson-Velsky worked in the Thermal Engineering Laboratory of the USSR Academy of Sciences (now the Institute for Theoretical and Experimental Physics). As a mathematician competent in computational mathematics, he worked productively together with physicists on the problem of building an efficient and reliable nuclear reactor and on the analysis of particle tracks in the bubble chamber of an accelerator, investigated models in nuclear physics, and so on [8]–[10]. This period was a turning point in his life: he became very interested in the broad possibilities and unusual (from the standpoint of a pure mathematician) problems in the efficient use of computing machines.

He was present at the emergence of at least two directions which play very important roles in modern computer science. The first concerns algorithms with polynomial run-time estimates (in what follows, polynomial-time algorithms). Up

[2]This is possible only in the case of a non-separable ring ([5], p. 58).

to about 1970 only a few dozen such algorithms were known, whereas by now theorists have developed many thousands of such algorithms, forming a basis for practical applications of computer science. Adelson-Velsky can be regarded as one of the founders of this area of study.

In this direction he and his students carried out a number of brilliant investigations. First of all, mention should be made of *balanced* trees [11], which he invented together with E. M. Landis and which are now known as AVL (Adelson-Velsky and Landis) trees. Originally intended to organize a rapid search for repeated positions in games, AVL trees became an example in computer science world-wide of a non-trivial effective data structure which changes as the data itself changes. They opened the way for a whole series of methods for organizing dynamic data structures such as splay trees, segment trees, Fenwick trees, and others. Today many dozens if not hundreds of dynamic structures are widely used in computer science.

The problem of a rapid search is well illustrated in terms of looking up a word in an ordinary dictionary containing n words. Finding a given word in such a dictionary requires at most $C \log n$ operations (the constant C depends on the model of computations and on the maximum possible word length). A new problem arises: how to organize a quick lookup in a dictionary whose content is constantly changing.

This problem was solved by Adelson-Velsky and Landis in their paper [11], now regarded as a classic in computer science. It turns out that words in a constantly growing dictionary should not be ordered linearly (in lexicographical order), as is usually done, but rather should be organized as a binary tree with a natural ordering: the right subtree goes up and the left subtree goes down. The tree should be *balanced*, that is, the heights of the left and right child subtrees of any node should differ by at most one. Lookup of a word in such a tree with n nodes also requires $C \log n$ operations. But what is most important is that both deletion and insertion in such a tree require the same number of operations. However, when a new word is inserted and takes its place in the naturally ordered tree (which requires $C \log n$ operations), the tree may become unbalanced: the height of the right subtree rooted at some 'unbalanced' node may differ from the height of the left subtree by more than one. The AVL algorithm in this case performs a rebalancing of the tree in a neighbourhood of the 'unbalanced' node with a finite (independent of n) number of operations in such a way that the tree becomes balanced again.

Thus, in a balanced dictionary, it is possible to organize quick lookup, insertion, or deletion of a word in such a way that the dictionary always remains balanced.

Beginning in the 1970s there was a rapid development of polynomial-time algorithms. The role of Adelson-Velsky in this process cannot be overestimated: he was the founder of the Moscow school of polynomial-time algorithms, one of the first if not the very first such school in the world, with contributions to international science that include many important achievements (see below). Its influence was based on the broad knowledge and deep understanding of the contemporary world science of polynomial-time algorithms taught by Adelson-Velsky to his students. He put particular emphasis on the use of dynamic data structures to speed up computations. For several years beginning in 1969 he conducted a seminar on algorithms. It was first a seminar for students in the Faculty of Mechanics and Mathematics at Moscow State University, and then a research seminar at the Institute of Control

Sciences of the USSR Academy of Sciences. This seminar was actually the centre of scientific activity in polynomial-time algorithmics in Moscow.

His school was distinguished by an innovative method for estimating the algorithm run-time and counting the number of operations on data structures. At that time the generally accepted approach to estimation of the run-time of an algorithm was based on estimating the maximum computation time during one iteration multiplied by the maximum number of iterations. In some complicated cases this method yields estimates that are too high, and this complicates the analysis and thereby hinders the development of new faster algorithms. Adelson-Velsky invented the much more powerful *distributional method* for estimating the run-time. The cost (execution time) of each substantive operation, or part of it, is distributed to ('put to the account of') one of the related elements of one of the base sets. The total time is calculated as the total sum over all elements of all base sets. The role of the base sets may be played by the set of iterations, the set of nodes or edges of the graph being investigated, and so on.

An example of a dynamic data structure with operation time that can be estimated using the distributional method is the construction of the levelled inquiry for shortest paths proposed by E. A. Dinic in his algorithm to find a maximum flow in a network [12]. It is organized as a version of the standard breadth-first search (BFS) strategy without discard of duplicating potentially appropriate edges. With the use of levelled inquiry the algorithm finds paths which augment the flow by an order of magnitude faster than all other methods. Although it might be very time-consuming to update the levelled list at one particular iteration, the total amount of time needed to update the list in the entire course of its use does not exceed the amount of time needed to construct it.

Today the distributional method of analysis is widely used in polynomial-time algorithmics, but regrettably as folklore, without mention of its inventor. Adelson-Velsky was the first to use it, and therefore in the 1960s and early 1970s this method was almost an exclusive art of the Moscow school of algorithms. In the mid-1970s a somewhat similar method for analysis of data structure efficiency was employed in some papers by Western scientists (see, for instance, [13]); this method was first presented in detail only in 1985 [14], under the name of amortized analysis.

Adelson-Velsky's students, with whom he shared his powerful tools for analyzing algorithms, have obtained and published very significant results in computer science such as 'the Method of Four Russians' [15], the ordering algorithm without a working memory [16], one of the first polynomial-time algorithms for solving the transportation problem [17], an algorithm [12] with an estimate [18] which is equivalent to the Hopcroft–Karp algorithm that finds a system of representatives of sets, pioneering algorithms for analyzing isomorphism of graphs [19] (for the related theory see the book [20] edited by B. Yu. Weisfeiler), and others. The names of their authors — either formal or informal students of Adelson-Velsky — are now widely known all over the world: V. L. Arlazarov, Weisfeiler, Dinic, M. V. Donskoy, A. V. Karzanov, M. A. Kronrod, A. A. Leman, P. A. Pevzner (bioinformatician), I. A. Faragev, B. V. Cherkasskii, and others. Results of Adelson-Velsky's students on flow algorithms were presented in the book [21], and are still necessarily included in all textbooks on algorithms.

The originality of the methods developed by Adelson-Velsky, which were far ahead of their time, is illustrated by the fact that some of the results obtained in the Moscow school of algorithms were certainly not immediately understood by Western scientists. Examples include AVL trees and Dinic's flow algorithm (a history of the failure to understand the latter can be found in [22]).

Another important and very fruitful area of his activity was 'artificial intelligence' or, as it was called at that time, 'heuristic programming'. First and foremost, we should mention here his years of work on the development of a chess program which was the world's best for about fifteen years. In 1974 the chess program "Kaissa" won the first world computer chess championship in Stockholm. This program served as a basis for elaboration and refinement of many general methods for information processing such as recursive sorting, pruning on the basis of formal considerations (for example, the method of bounds and valuations, known as alpha-beta pruning) and informal considerations (for example, forced variants), and organization of inquiries (search trees, hash tables, and so on).

Also, the following fundamental problem in the theory of games with perfect information was studied. It was shown by Zermelo long ago that if such a game is represented in the form of a tree whose leaves are marked with evaluations (the game payoffs), then there exist an algorithm (a minimax procedure) that evaluates the initial position (the root of the tree) and an optimal strategy for each of the players which leads to this evaluation. This means, in particular, that for any chess position the outcome of the game is predetermined, provided that the players follow their optimal strategies.

However, no chess program actually goes as far as the final position in its calculations, but rather confines itself to a search over some subtree of the game to a certain depth. The leaf (final) positions of this subtree are evaluated heuristically. It might seem evident that — for a 'reasonable' evaluation procedure — the deeper the subtree the better the performance of the program. But this is not always the case.

In [23] Adelson-Velsky, V. P. Akimov, and Arlazarov demonstrated that there are definite limits for the 'reasonableness' of the heuristic evaluation (understood as the probability of it being close to the true evaluation) within which it makes sense to make the search deeper. In other words, if this probability is sufficiently high, then it increases as the search depth increases; otherwise, it decreases. It is interesting to note that in some simplest cases the calculated boundary of 'reasonableness' is far from the intuitive boundary.

Another important result obtained during the development of the chess program was the theory and practical implementation of a 'geometric decomposition' of the search, or in modern terms, a 'retrieval of knowledge' from the search.

One of the main problems that hinder the deepening of the search is the presence of a huge number of repeating subtrees, the traversal of which takes a major portion of the total amount of time needed to make a move.

Adelson-Velsky and Donskoy (see, for instance, [24]) proposed associating with each subtree A rooted at a position P_i a system $Q(P_i)$ of squares of the chessboard with the following property: if a position P_j is the root of another subtree, but the set of squares involved in the change of the position P_j as compared with P_i does not intersect $Q(P_i)$, then the subtree A appears exactly as it is at the position P_j

and gives the same evaluation as it does at the position P_i. Thus, the information obtained during the search at the position P_i can also be used at other positions.

Adelson-Velsky's D.Sc. thesis "The method of structural graphs for problems of discrete optimization" was defended in 1974 at the Institute of Control Sciences (Moscow). It included results obtained in the course of his work on the chess program, as well as some results on flow algorithms and network planning and design.

He also made significant contributions to developments in the mathematical education of school-children. He was a coauthor of the first edition of *Selected problems and theorems of elementary mathematics*, a collection of problems of olympiad type which was intensively used for many years in the work of extracurricular mathematical groups [25]. The mid-1960s was a happy period for school mathematical education in the USSR. Mathematicians had an opportunity to teach children not only in select groups, but also at secondary schools. In addition to the four university-based specialized boarding schools, some 'ordinary' municipal schools became specialized in mathematics and programming. Together with A. S. Kronrod and N. N. Konstantinov, he taught at one of the first mathematical schools in the Soviet Union: Moscow Secondary School no. 7. He once formulated the reason for his activities of this kind: "I want to help children become free people". Following the tradition of extracurricular mathematical groups, they practiced 'problem-based' teaching, the subject being the elements of calculus (or the theory of functions of a real variable, under the influence of A. S. Kronrod, who was a student of Luzin). Adelson-Velsky realized his approach in 1964–1966 together with his younger colleagues Leman, L. Limanov, M. Yakobson, and I. I. Yudina, a distinguished teacher of school mathematics. One of the graduates of that mathematical class remembered: "Adelson-Velsky taught us to seek the truth for the sake of truth, though he would never have formulated it that way: pomposity was absolutely alien to his nature". The tradition which was established at that time with the participation of Adelson-Velsky, and which is maintained in our day first and foremost by students of Konstantinov's students, is very important for the role of mathematics in modern Russian secondary education.

It was a strong conviction of Adelson-Velsky that commerce is adverse to programming: "The grandiose science of mathematics is out of keeping with commerce. And if it is proprietary right that comes to the forefront, then the spirit of collective creative work fades away and the greatest creator — the team of scientists — loses its power".

The book *Discrete mathematics for engineers*, written with O. P. Kuznetsov (the first edition was published in 1980), played an important role in the establishment of the modern mathematical background for the computer sciences at technical universities. At that time it was the only book in Russian that contained a systematic presentation of all the fundamental areas of discrete mathematics, including the main concepts of the theory of computational complexity, which was then little known to the academic staff of technical universities. It is not surprising that, in spite the fact that 25 000 copies were printed, the book became a rarity in 3 or 4 years. An English translation of the first edition was published in 1985. In 1988 the second, augmented edition was issued, containing, in particular, the first textbook presentation of Khachiyan's polynomial-time algorithm for solving the linear-programming problem.

Adelson-Velsky's last paper was written in 2002 [26]. At that time he was a professor at Bar-Ilan University in Israel, where he moved in 1992.

On April 26, 2014, after a long struggle with a serious illness, Georgy Maksimovich Adelson-Velsky died in Tel Aviv. He was 92 years old.

We shall always remember him as a man of sparkling talents, modesty, diligence, and uncompromising honesty.

V. L. Arlazarov, E. A. Dinitz, Yu. S. Ilyashenko, A. V. Karzanov, S. M. Karpenko, A. A. Kirillov, N. N. Konstantinov, M. A. Kronrod, O. P. Kuznetsov, L. B. Okun', P. A. Pevzner, A. L. Semenov, I. A. Faradzhev, B. V. Cherkasskii, and A. G. Khovanskii

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